Sietse Buijsman

Second Final exam: **Electrodynamics of Radiation Processes,**
13th April 2018, 9-12
The exam consists of 100 points in total.

Write your name and student ID number on every page.
Make certain to clearly label which answer is which on your exam papers.

Only calculators can be used. **No laptops, tablets, iPads, or other internet devices are allowed. Also no books or notes are allowed.**

Explain clearly all of the steps that you use to derive all your results. If you are not sure of a particular step make an estimate, be clear you doing this, and continue.

Make certain that your handwriting is readable to someone besides yourself.

Useful constants:

$$
\begin{aligned}
c &= 3 \times 10^{8}\,\mathrm{m\,s^{-1}} \\
k &= 1.38 \times 10^{-23}\,\mathrm{m^2\,kg\,s^{-2}\,K^{-1}} \\
&= 8.6 \times 10^{-5}\,\mathrm{eV\ K^{-1}} \\
h &= 6.63 \times 10^{-34}\,\mathrm{m^2\,kg\,s^{-1}} \\
1\,\mathrm{parsec} &= 3 \times 10^{16}\,\mathrm{m}
\end{aligned}
$$

1. **Radiative Transfer** [20 pts]

   (a) [**4 pts**] How is a solid angle defined?

   (b) [**6 pts**] Assuming we only see the surface of a thick spherical cloud, with a constant intensity (see Figure 1), calculate the flux in terms of intensity if the distance of the cloud, r, from the observer is much greater than the intrinsic radius of the cloud, R ($r \gg R$). How is the result different for r = R?

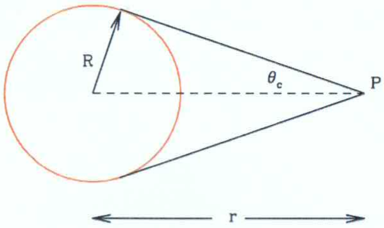

Figure 1: An observer at P sees a spherical source as a disc of angular radius θ_c, where the intrinsic radius of the source is R, and it is at distance r from an observer at P.

   (c) [**3 pts**] Explain what is meant by optical depth.

   (d) [**5 pts**] Explain the radiative transfer equation:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

   (e) [**2 pts**] Give the solution to this equation, giving a general expression for specific intensity, in the case of emission only.

2. **Blackbody Radiation** [20 pts]

   (a) [**5 pts**] Explain what is a Blackbody, and are the properties of the observed radiation. Give an example of an observable Blackbody.

   (b) [**5 pts**] Show that there is monotonicity with temperature for the Planck law:

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp(\frac{h\nu}{kT}) - 1}$$

   (c) [**10 pts**] Consider a spherical cloud with particles emitting Blackbody radiation. These particles all have the same temperature T = 4×10^3K. The cloud has a diameter of 0.1 pc. At frequency $\nu_0 = 1.3 \times 10^{15}$ Hz the particles have an absorption coefficient $\alpha_\nu = 7 \times 10^{-12}\text{m}^{-1}$.
   What is the total isotropic luminosity at frequency ν_0 emitted by this cloud, assuming that it is optically thin?

3. **Synchrotron Radiation** [20 pts]

   (a) **[5 pts]** Explain how Synchrotron radiation can occur? Give an example of an astrophysical (or terrestrial) source.

   (b) **[7 pts]** Why is Synchrotron radiation frequently polarised? and how does the degree of polarisation depend upon the distribution index? and so what is the degree of polarisation for a typical power law spectrum?

   (c) **[3 pts]** Sketch the single particle spectrum of synchrotron radiation, and make another sketch to show how a population of particles creates a power law spectrum.

   (d) **[5 pts]** Show how the energy of an ultra-relativistic electron that emits synchrotron radiation will decrease with time according to:

$$\gamma = \gamma_0(1 + A\gamma_0 t)^{-1}, \text{where } \mathrm{A} = \frac{2\mathrm{e}^4\mathrm{B}_\perp^2}{3\mathrm{m}^2\mathrm{c}^5}$$

   where γ_0 is the initial value of γ and $B_\perp = B \sin\alpha$.

4. **Bremsstrahlung** [20 pts]

   (a) **[5 pts]** Explain the typical origin of Bremsstrahlung (also called free-free) radiation. Give an example of a known source of Bremsstrahlung.

   (b) **[5 pts]** Explain the terms in the Bremsstrahlung absorption equation:

$$\alpha_\nu^{ff} = 3.7 \times 10^8 T^{-1/2} Z^2 n_e n_i \nu^{-3} (1 - e^{-h\nu/kT})\,\bar{g}_{ff}$$

   Explain what happens when $\mathrm{h}\nu \gg \mathrm{kT}$.

   (c) **[4 pts]** Make a sketch to explain what is meant by the importance of the impact parameter, b, for Bremsstrahlung radiation. What determines the lower limit to this parameter?

   (d) **[6 pts]** Using the X-ray Bremsstrahlung spectrum of the central region of the Virgo cluster of galaxies shown in Figure 2, estimate the temperature of the emitting gas.

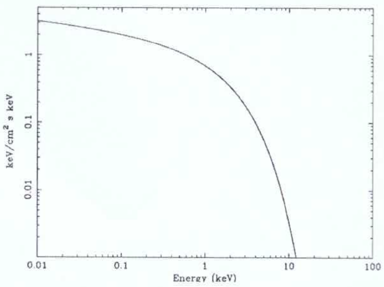

Figure 2: The X-ray spectrum of the central region of the Virgo cluster.

5. **Compton Scattering** [**20 pts**]

(a) [**8 pts**] Explain the basic principles of Compton scattering, and the properties of the resulting radiation. Describe the basic physical processes that are involved.

(b) [**4 pts**] What is inverse Compton scattering? why are relativistic effects now important?

(c) [**4 pts**] What is the Compton y-parameter, and how is it defined? What does $y \gg 1$ mean?

(d) [**4 pts**] Using Figure 3, if it helps you, explain what is the Sunyaev-Zeldovich effect.

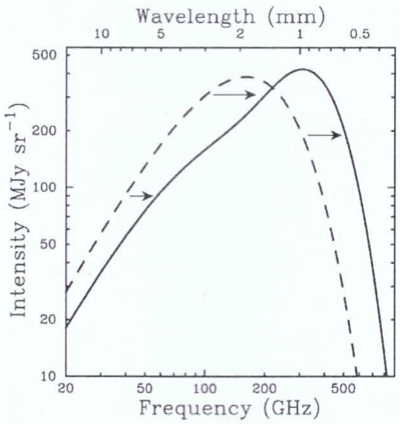

Figure 3: The cosmic microwave background spectrum, undistorted (dashed line) and distorted by the Sunyaev-Zeldovich effect (SZE) (solid line).